ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_0^{\frac{\pi}{6}} \frac{tanx}{(1+cosx)^2}dx$$

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Solution by Tapas Das-India

$$\int \frac{tanx}{(1+cosx)^2}dx = \int \frac{sinx}{cosx(1+cosx)^2}dx \overset{tanx=u}{=} -\int \frac{du}{u(1+u)^2} =$$

$$= -\int \frac{(1+u)-u}{u(1+u)^2}du = -\int \frac{du}{u(1+u)} + \int \frac{du}{(1+u)^2} =$$

$$= -\int \frac{(1+u)-u}{u(1+u)}du - \frac{1}{1+u} = -\int \frac{1}{u}du + \int \frac{du}{1+u} - \frac{1}{1+u} =$$

$$= -\log|u| + \log|1+u| - \frac{1}{1+u} = \log\left|\frac{1+u}{u}\right| - \frac{1}{1+u} = \log\left|\frac{1+cosx}{cosx}\right| - \frac{1}{1+cosx}$$

$$\int_0^{\frac{\pi}{6}} \frac{tanx}{(1+cosx)^2}dx = \left[\log\left|\frac{1+cosx}{cosx}\right| - \frac{1}{1+cosx}\right]_0^{\frac{\pi}{6}} = \log\left(\frac{2+\sqrt{3}}{2}\right) + \frac{1}{2} - \frac{2}{2+\sqrt{3}}$$