ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_0^{\frac{\pi}{6}} \frac{1}{(1+\cos(x))^3}\,dx$$

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$$\int_0^{\frac{\pi}{6}} \frac{1}{(1+\cos(x))^3}dx = \int_0^{\frac{\pi}{6}} \frac{1}{8\cos^6\left(\frac{x}{2}\right)}dx = \frac{1}{8}\int_0^{\frac{\pi}{6}} \frac{1}{\cos^4\left(\frac{x}{2}\right)}\frac{dx}{\cos^2\left(\frac{x}{2}\right)} =$$

$$(\text{Let } \tan\left(\frac{x}{2}\right) = t \rightarrow \ dt = \frac{1}{2}.\frac{dx}{\cos^2\left(\frac{x}{2}\right)})$$

$$(\frac{1}{\cos^4\left(\frac{x}{2}\right)} = \left(1+\tan^2\left(\frac{x}{2}\right)\right)^2 = (1+t^2)^2 = 1+2t^2+t^4)$$

$$= \frac{1}{8}\int_0^{2-\sqrt{3}} (1+2t^2+t^4).2dt = \frac{1}{4}\left(t+\frac{2t^3}{3}+\frac{t^5}{5}\right)\Bigg|_0^{2-\sqrt{3}} =$$

$$= \frac{1}{4}\left((2-\sqrt{3})+\frac{2}{3}(2-\sqrt{3})^3+\frac{1}{5}(2-\sqrt{3})^5\right) =$$

$$= \frac{1}{4}\left(2+\frac{52}{3}+\frac{362}{5}-\sqrt{3}-10\sqrt{3}-\frac{209}{5}\sqrt{3}\right) = \frac{1}{4}\left(\frac{1376}{15}-\frac{264}{5}\sqrt{3}\right) = \frac{344}{15}-\frac{66}{5}\sqrt{3}$$