ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_0^{\frac{\pi}{6}} \frac{1}{(1+\sin(x))^2}dx$$

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$$\int_0^{\frac{\pi}{6}} \frac{1}{(1+\sin(x))^2}dx = \int_0^{\frac{\pi}{6}} \frac{1}{\left(1+\cos\left(\frac{\pi}{2}-x\right)\right)^2}dx \overset{\frac{\pi}{2}-x\to x}{\triangleq}$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{(1+\cos(x))^2}dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{4cos^4\left(\frac{x}{2}\right)}dx = \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{cos^2\left(\frac{x}{2}\right)} \cdot \frac{dx}{2cos^2\left(\frac{x}{2}\right)} =$$

$$= \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(1+tan^2\left(\frac{x}{2}\right)\right)\frac{dx}{2cos^2\left(\frac{x}{2}\right)} \overset{\tan\left(\frac{x}{2}\right)=t\to\ dt=\frac{dx}{2cos^2\left(\frac{x}{2}\right)}}{\triangleq}$$

$$= \frac{1}{2}\int_{\frac{\sqrt{3}}{3}}^{1}(1+t^2)dt = \frac{1}{2}\left(t+\frac{t^3}{3}\right)\Bigg|_{\frac{\sqrt{3}}{3}}^{1} = \frac{1}{2}\left(\frac{4}{3}-\frac{8\sqrt{3}}{27}\right) = \frac{2}{3}-\frac{4\sqrt{3}}{27}$$