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In any triangle ABC with the area F the following inequality holds :

$$\frac{a^4+b^4}{m_c^2}+\frac{b^4+c^4}{m_a^2}+\frac{c^4+a^4}{m_b^2}\ge\frac{32\sqrt{3}}{3}F$$

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Solution by Soumava Chakraborty-Kolkata-India

$$LHS=\sum_{cyc}\frac{b^4+c^4}{m_a^2}\ge\sum_{cyc}\frac{(b^2+c^2)^2}{2m_a^2}\overset{\text{Bergstrom}}{\ge}\frac{\left(\sum_{cyc}(b^2+c^2)\right)^2}{2\sum_{cyc}m_a^2}=$$

$$=\frac{2\left(\sum_{cyc}a^2\right)^2}{\frac{3}{4}\sum_{cyc}a^2}=\frac{8}{3}.\sum_{cyc}a^2\overset{\text{Ionescu-Weitzenbock}}{\ge}\frac{8}{3}.4\sqrt{3}.F=\frac{32\sqrt{3}}{3}F\text{ and so,}$$

$$\frac{a^4+b^4}{m_c^2}+\frac{b^4+c^4}{m_a^2}+\frac{c^4+a^4}{m_b^2}\ge\frac{32\sqrt{3}}{3}F\ \forall\ \Delta\ ABC,$$

$''=''$ iff Δ ABC is equilateral (QED)