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In $\triangle ABC$ the following relationship holds:

$$2s + a^3 + b^3 + c^3 \geq 8\sqrt{3}F$$

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Solution by Tapas Das-India

$$\begin{aligned} 2s + a^3 + b^3 + c^3 &= a + b + c + a^3 + b^3 + c^3 \stackrel{AM-GM}{\geq} 6\sqrt[6]{(abc)^4} = \\ &= 6((abc)^2)^{\frac{1}{3}} \stackrel{Carlitz}{\geq} 6 \times \frac{4F}{\sqrt{3}} = 8\sqrt{3}F \end{aligned}$$

Equality holds for an equilateral triangle with: $a = b = c = 1$.