ROMANIAN MATHEMATICAL MAGAZINE

If $x, y \geq 0$ and $x + y = 6$ then in ΔABC the following relationship holds:

$$(2s + a^x + b^y + c^x)(2s + a^y + b^x + c^y) \geq 192F^2$$

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Solution by Tapas Das-India

$$(2s + a^x + b^y + c^x) = (a + b + c + a^x + b^y + c^x) \overset{AM-GM}{\geq} 6\sqrt[6]{abca^x b^y c^x}$$

$$Similarly: (2s + a^y + b^x + c^y) \geq 6\sqrt{abca^y b^x c^y}$$

$$(2s + a^x + b^y + c^x)(2s + a^y + b^x + c^y) \geq 6\sqrt[6]{abca^x b^y c^x} \times 6\sqrt{abca^y b^x c^y} =$$

$$= 36\sqrt[6]{(abc)^2(abc)^{x+y}} \overset{x+y=6}{=} 36(abc)^{\frac{4}{3}} = 36((abc)^2)^{\frac{2}{3}} \overset{Carlitz}{\geq} 36.\left(\frac{4F}{\sqrt{3}}\right)^2 = 192F^2$$

Equality holds for: $a = b = c = 1, x = y = 3.$