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If $x,y,z>0$ and M is an interior point in triangle ABC with the are F and d_a,d_b,d_c are the distances of point M to the sides BC,CA,AB then:

$$\frac{x^2a^3}{(y+z)^2d_a}+\frac{y^2b^3}{(x+z)^2d_b}+\frac{z^2c^3}{(y+x)^2d_c}\geq 6F$$

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$$WLOG\ \ 0<x\leq y\leq z\rightarrow\frac{x}{y+z}\leq\frac{y}{x+z}\leq\frac{z}{x+y}$$

$$and\ \ a\leq b\leq c\rightarrow\frac{1}{d_a}\leq\frac{1}{d_b}\leq\frac{1}{d_c}$$

$$\sum_{cyc}\frac{x^2a^3}{(y+z)^2d_a}\overset{Chebyshev}{\geq}\frac{1}{3}\sum_{cyc}\frac{x^2}{(y+z)^2}.\sum_{cyc}\frac{a^3}{d_a}=\frac{1}{3}\sum_{cyc}\frac{x^2}{(y+z)^2}.\sum_{cyc}\frac{a^4}{ad_a}\overset{Bergstrom}{\geq}$$

$$\geq\frac{1}{3}.\frac{\left(\sum_{cyc}x\right)^2}{2\left(\sum_{cyc}x^2+\sum_{cyc}xy\right)}.\frac{\left(\sum_{cyc}a^2\right)^2}{\sum_{cyc}ad_a}\geq\frac{1}{3}.\frac{\left(\sum_{cyc}x\right)^2}{2\left(\sum_{cyc}xy+\sum_{cyc}xy\right)}.\frac{\left(4\sqrt{3}.F\right)^2}{2.F}=$$

$$=\frac{1}{3}.\frac{\left(\sum_{cyc}x\right)^2}{\sum_{cyc}xy}.24F\geq\frac{1}{3}.\frac{3}{4}.24.F=6F$$

Equality holds if:

$$a=b=c\ \ and\ \ x=y=z,\qquad d_a=d_b=d_c=r$$