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Let M be an interior point in triangle ABC with the are F and d_a, d_b, d_c the distances of point M to the sides BC, CA, AB. Prove that:

$$\frac{a^3}{d_a}+\frac{b^3}{d_b}+\frac{c^3}{d_c}\geq 24F$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{a^3}{d_a}+\frac{b^3}{d_b}+\frac{c^3}{d_c}=\frac{a^4}{ad_a}+\frac{b^4}{bd_b}+\frac{c^4}{cd_c}\overset{Bergstrom}{\geq}$$

$$\geq\frac{(a^2+b^2+c^2)^2}{ad_a+bd_b+cd_c}\overset{Ionescu-Weitzenbock}{\geq}\frac{\left(4\sqrt{3}F\right)^2}{2F}=24F$$

Equality holds for:

$$a=b=c \text{ and } d_a=d_b=d_c=r$$