ROMANIAN MATHEMATICAL MAGAZINE

Let $u,v>0$ and ABC a triangle with the area F. Prove that:

$$\left(\left(ua^2+vm_b^2\right)^2+2\right)\left((ub^2+vm_c^2)^2+2\right)\left((uc^2+vm_a^2)^2+2\right)\ge 9(4u+3v)^2F^2$$

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Let's use Arkady Alt's formula to prove:

$$x,y,z,t>0$$

$$(x^2+t^2)(y^2+t^2)(z^2+t^2)\ge\frac{3}{4}t^4(x+y+z)^2$$

$$If\ \ x=ua^2+vm_b^2,\quad y=ub^2+vm_c^2,\quad z=uc^2+vm_a^2,\quad t=\sqrt{2}$$

Then:

$$\left(\left(ua^2+vm_b^2\right)^2+2\right)\left((ub^2+vm_c^2)^2+2\right)\left((uc^2+vm_a^2)^2+2\right)\ge$$

$$\frac{3}{4}\left(\sqrt{2}\right)^4\left(\left(ua^2+vm_b^2\right)+\left(ub^2+vm_c^2\right)+\left(uc^2+vm_a^2\right)\right)^2=$$

$$3\left((a^2+b^2+c^2)u+\left(m_a^2+m_b^2+m_c^2\right)v\right)^2=$$

$$3\left((a^2+b^2+c^2)u+\frac{3}{4}(a^2+b^2+c^2)v\right)^2\overset{Ionescu-Weitzenbock\ (a^2+b^2+c^2)\ge4\sqrt{3}F}{\ge}$$

$$\ge3\left(4\sqrt{3}.Fu+\frac{3}{4}.4\sqrt{3}.Fv\right)^2=9(4u+3v)^2F^2$$

Equality holds for $a=b=c$ and $t=\sqrt{2}$