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Let M be an interior point in triangle ABC with the area F and d_a, d_b, d_c the distances of point M to the sides BC, CA respectively AB. Prove that:

$$\frac{ab^2}{d_a}+\frac{bc^2}{d_b}+\frac{ca^2}{d_c}\geq 24F$$

Proposed by D.M.Bătinețu-Giurgiu, Claudia Nănuți-Romania

Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{ab^2}{d_a}+\frac{bc^2}{d_b}+\frac{ca^2}{d_c}=\frac{ab^2}{\frac{2F_a}{a}}+\frac{bc^2}{\frac{2F_b}{b}}+\frac{ca^2}{\frac{2F_c}{c}}=\frac{(ab)^2}{2F_a}+\frac{(bc)^2}{2F_b}+\frac{(ac)^2}{2F_c}\overset{Bergstrom}{\geq}$$

$$\geq\frac{(ab+bc+ac)^2}{2(F_a+F_b+F_c)}=\frac{(ab+bc+ac)^2}{2F}\overset{Gordon}{\geq}\frac{\left(4\sqrt{3}F\right)^2}{2F}=24F$$

$$Equality\ \ holds\ \ for:\ \ a=b=c.$$