ROMANIAN MATHEMATICAL MAGAZINE

If $x, y > 0$, $x + y = 2$ then in ΔABC the following relationship holds:

$$(a^x.b^y + 2)(b^x.c^y + 2)(c^x.a^y + 2) \geq 36\sqrt{3}F$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$Let's\ \ use\ \ Arkady\ \ Alt's\ \ formula\ \ to\ \ prove:$$

$$m, n, p \geq 0\ ,\ t > 0$$

$$(m^2 + t^2)(n^2 + t^2)(p^2 + t^2) \geq \frac{3}{4}t^4(m + n + p)^2$$

$$Here:\ m^2 = a^x.b^y;\ n^2 = b^x.c^y;\ p^2 = c^x.a^y\ ,\quad t^2 = 2$$

$$Then:$$

$$(a^x.b^y + 2)(b^x.c^y + 2)(c^x.a^y + 2) \overset{Arkady\ \ Alt}{\geq} \frac{3}{4}.4.\left(\sqrt{a^x.b^y} + \sqrt{b^x.c^y} + \sqrt{c^x.a^y}\right)^2 \overset{A-G}{\geq}$$

$$\geq 3\left(3.\left(a^{x+y}.b^{x+y}.c^{x+y}\right)^{\frac{1}{6}}\right)^2 = 27(abc)^{\frac{2}{3}} =$$

$$= 27.(4.R.F)^{\frac{2}{3}} = 27.(16.R^2.F^2)^{\frac{1}{3}} \overset{F \leq \frac{\sqrt{27}}{4}R^2}{\geq} 27.\left(16.\frac{4F}{\sqrt{27}}.F^2\right)^{\frac{1}{3}} = \frac{108}{\sqrt{3}}.F = 36\sqrt{3}F$$

$$Equality\ \ holds\ \ for\ \ a = b = c\ ,\quad and\ \ t = \sqrt{2}.$$