ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$ then in ΔABC the following relationship holds:

$$\frac{x}{(y+z)h_a^2}+\frac{y}{(z+x)h_b^2}+\frac{z}{(x+y)h_c^2}\geq\frac{\sqrt{3}}{2F}$$

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Solution by Tapas Das-India

$$\frac{x}{(y+z)h_a^2}+\frac{y}{(z+x)h_b^2}+\frac{z}{(x+y)h_c^2}=$$

$$=\sum\frac{x}{(y+z)\frac{4F^2}{a^2}}=\frac{1}{4F^2}\sum\frac{x}{y+z}a^2\overset{Tsintsifas}{\geq}\frac{1}{4F^2}.2\sqrt{3}F=\frac{\sqrt{3}}{2F}$$

Equality holds for an equilateral triangle and x=y=z.