ROMANIAN MATHEMATICAL MAGAZINE

If $x, y \geq 0$, $x + y > 0$ then in ΔABC the following relationship holds:

$$\frac{a^3}{xr + yh_a} + \frac{b^3}{xr + yh_b} + \frac{c^3}{xr + yh_c} \geq \frac{24F}{x + 3y}$$

Proposed by D.M.Bătinețu-Giurgiu, Claudia Nănuți-Romania

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc} \frac{a^3}{xr + yh_a} = \sum_{cyc} \frac{a^3}{xr + y.\frac{2F}{a}} =$$

$$= \sum_{cyc} \frac{a^4}{axr + 2sry} = \frac{1}{r} . \sum_{cyc} \frac{a^4}{ax + 2sy} \overset{Cauchy-Schwarz}{\geq}$$

$$= \frac{1}{r} . \frac{(\sum a)^4}{3^2 . (x \sum a + 6sy)} = \frac{(\sum a)^4}{9r . 2s(x + 3y)} =$$

$$= \frac{1}{r} . \frac{(2s)^3}{9(x + 3y)} = \frac{8s^2 . s}{9r(x + 3y)} \overset{Mitrinovic}{\geq} \frac{8 . 27 . r^2 . s}{9r(x + 3y)} = \frac{24sr}{x + 3y} = \frac{24F}{x + 3y}$$

Equality holds for $a = b = c$.