ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{1}{(b+c)h_a}+\frac{1}{(a+c)h_b}+\frac{1}{(b+a)h_c}\ge\frac{3}{4F}$$

Proposed by D.M.Bătinețu-Giurgiu, Dan Nănuți-Romania

Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{1}{(b+c)h_a}+\frac{1}{(a+c)h_b}+\frac{1}{(b+a)h_c}=$$

$$=\frac{a}{(b+c).2F}+\frac{b}{(a+c).2F}+\frac{c}{(b+a).2F}=$$

$$=\frac{1}{2F}\left(\frac{a^2}{ab+ac}+\frac{b^2}{ab+bc}+\frac{c^2}{ac+bc}\right)\overset{Bergstrom}{\ge}$$

$$\ge\frac{1}{2F}\cdot\frac{(a+b+c)^2}{2(ab+bc+ac)}\ge\frac{1}{2F}\cdot\frac{9}{2}=\frac{3}{4F}$$

$$Equality\ \ holds\ \ for\ \ a=b=c.$$