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In ΔABC the following relationship holds:

$$a^2+b^2+c^2 \geq 6F+\frac{1}{4}\left(\left(a-\sqrt{3}b\right)^2+\left(b-\sqrt{3}c\right)^2+\left(c-\sqrt{3}a\right)^2\right)$$

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Solution by Tapas Das-India

$$\frac{1}{4}\left(\left(a-\sqrt{3}b\right)^2+\left(b-\sqrt{3}c\right)^2+\left(c-\sqrt{3}a\right)^2\right)=$$

$$=\frac{1}{4}\left((a^2+b^2+c^2)+3(a^2+b^2+c^2)-2\sqrt{3}(ab+bc+ca)\right)$$

$$=(a^2+b^2+c^2)-\frac{\sqrt{3}}{2}(ab+bc+ca)$$

$$We\ need\ to\ show:$$

$$(a^2+b^2+c^2)\geq 6F+\ (a^2+b^2+c^2)-\frac{\sqrt{3}}{2}(ab+bc+ca)$$

$$\frac{\sqrt{3}}{2}(ab+bc+ca)\geq 6F\ this\ is\ true\ \ since:$$

$$\frac{\sqrt{3}}{2}(ab+bc+ca)\overset{Gordon}{\geq}\frac{\sqrt{3}}{2}4\sqrt{3}F=6F$$

Equality holds for an equilateral triangle.