ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$ then in ΔABC the following relationship holds:

$$\frac{x^2}{(y+z)^2}+\frac{y^2}{(x+z)^2}+\frac{z^2}{(y+x)^2}+a^4+b^4+c^4\geq 4\sqrt{3}F$$

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Solution by Tapas Das-India

$$\frac{x^2}{(y+z)^2}+\frac{y^2}{(x+z)^2}+\frac{z^2}{(y+x)^2}+a^4+b^4+c^4=$$

$$=\frac{x^2}{(y+z)^2}+\frac{y^2}{(x+z)^2}+\frac{z^2}{(y+x)^2}+(a^2)^2+(b^2)^2+(c^2)^2\geq$$

$$\overset{CBS}{\geq}\frac{1}{3}\left(\sum\frac{x}{y+z}\right)^2+\frac{1}{3}(a^2+b^2+c^2)^2\overset{AM-GM}{\geq}\frac{2}{3}\sqrt{\left(\sum\frac{x}{y+z}\right)^2(a^2+b^2+c^2)^2}$$

$$\geq\frac{2}{3}\sum\frac{x}{y+z}(a^2+b^2+c^2)\overset{Nesbitt\ \&\ Ionescu-Weitzenbock}{\geq}\frac{2}{3}\cdot\frac{3}{2}4\sqrt{3}F=4\sqrt{3}F$$

Equality holds for an equilateral triangle and x=y=z.