

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\triangle ABC$  the following relationship holds:

$$\frac{2a^4 - a^2b^2 + 2b^2}{a^2 - ab + b^2} + \frac{2b^4 - c^2b^2 + 2c^2}{b^2 - cb + c^2} + \frac{2c^4 - a^2c^2 + 2a^2}{c^2 - ac + a^2} \geq 12\sqrt{3}F$$

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*Solution by Tapas Das-India*

$$\begin{aligned} 2a^4 - a^2b^2 + 2b^2 &= 2(a^4 + b^4) - a^2b^2 \stackrel{CBS}{\geq} 2 \cdot \frac{(a^2 + b^2)^2}{2} - (ab)^2 = \\ &= (a^2 + ab + b^2)(a^2 - ab + b^2) \quad (1) \end{aligned}$$

$$\begin{aligned} &\frac{2a^4 - a^2b^2 + 2b^2}{a^2 - ab + b^2} + \frac{2b^4 - c^2b^2 + 2c^2}{b^2 - cb + c^2} + \frac{2c^4 - a^2c^2 + 2a^2}{c^2 - ac + a^2} = \\ &= \sum \frac{2a^4 - a^2b^2 + 2b^2}{a^2 - ab + b^2} \stackrel{(1)}{\geq} \sum \frac{(a^2 + ab + b^2)(a^2 - ab + b^2)}{a^2 - ab + b^2} = \\ &= \sum (a^2 + ab + b^2) = 2(a^2 + b^2 + c^2) + (ab + bc + ca) \geq \end{aligned}$$

$$\geq 2(ab + bc + ca) + (ab + bc + ca) = 3(ab + bc + ca) \stackrel{Gordon}{\geq} 3 \cdot 4\sqrt{3}F = 12\sqrt{3}F$$

Equality holds for an equilateral triangle.