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If $m \geq 0$ then in ΔABC the following relationship holds:

$$\frac{1}{h_a^{m+1}} + \frac{a^{m+1}}{h_b^{m+1}} + \frac{b^{m+1}c^{m+1}}{h_c^{m+1}} \geq 2^{m+1}\left(\sqrt{3}\right)^{1-m}$$

Proposed by D.M.Bătinețu-Giurgiu, Claudia Nănuți-Romania

Solution by Tapas Das-India

$$\frac{1}{h_a^{m+1}} + \frac{a^{m+1}}{h_b^{m+1}} + \frac{b^{m+1}c^{m+1}}{h_c^{m+1}} = \frac{a^{m+1}}{(2F)^{m+1}} + \frac{(ab)^{m+1}}{(2F)^{m+1}} + \frac{(bc^2)^{m+1}}{(2F)^{m+1}} \overset{AM-GM}{\geq}$$

$$\geq \frac{3}{(2F)^{m+1}}\sqrt[3]{((abc)^2)^{m+1}} \overset{Carlitz}{\geq} \frac{\left(\sqrt{3}\right)^2}{(2F)^{m+1}} \cdot \left(\frac{4F}{\sqrt{3}}\right)^{m+1} = 2^{m+1}\left(\sqrt{3}\right)^{1-m}$$

Equality holds for an equilateral triangle & m=0.