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If $m \geq 0$ then in ΔABC the following relationship holds:

$$\frac{a^m b^m}{h_a h_b} + \frac{b^m c^m}{h_b h_c} + \frac{c^m a^m}{h_c h_a} \geq 4^m \left(\sqrt{3}\right)^{1-m} F^{m-1}$$

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Solution by Tapas Das-India

$$\frac{a^m b^m}{h_a h_b} + \frac{b^m c^m}{h_b h_c} + \frac{c^m a^m}{h_c h_a} = \sum \frac{a^m b^m}{h_a h_b} = \sum \frac{a^m b^m}{\frac{2F}{a} \cdot \frac{2F}{b}} = \frac{1}{4F^2} \sum (ab)^{m+1} \overset{CBS}{\geq}$$

$$\geq \frac{1}{4F^2} \cdot \frac{1}{3^m} (ab + bc + ca)^{m+1} \overset{Gordon}{\geq} \frac{1}{4F^2} \cdot \frac{1}{3^m} \left(4\sqrt{3}F\right)^{m+1} = 4^m \left(\sqrt{3}\right)^{1-m} F^{m-1}$$

Equality holds for an equilateral triangle & m=0.