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In any triangle $T = ABC$, with the area F we denote:

$$u(T) = \frac{a}{h_a}ctg(A) + \frac{b}{h_b}ctg(B) + \frac{c}{h_c}ctg(C)$$

Prove that:

$$\left(a^4 + u(T)\right)\left(b^4 + u(T)\right)\left(c^4 + u(T)\right) \geq 144F^2$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$u(T) = \frac{a}{h_a}ctg(A) + \frac{b}{h_b}ctg(B) + \frac{c}{h_c}ctg(C) = \frac{a}{\frac{2F}{a}}ctg(A) + \frac{b}{\frac{2F}{b}}ctg(B) + \frac{c}{\frac{2F}{c}}ctg(C) =$$

$$= \frac{a\left(2Rsin(A)\right).\cos(A)}{2Fsin(A)} + \frac{b\left(2Rsin(B)\right).\cos(B)}{2Fsin(B)} + \frac{c\left(2Rsin(C)\right).\cos(C)}{2Fsin(C)} =$$

$$= \frac{R}{F}(a.\cos(A) + b.\cos(B) + c.\cos(C)) = \frac{R}{F}.\frac{2F}{R} = 2$$

$$u(T) = 2$$

$$\left(a^4 + u(T)\right).\left(b^4 + u(T)\right).\left(c^4 + u(T)\right) = (a^4 + 2).(b^4 + 2).(c^4 + 2) \overset{Arkady\ Alt}{\geq}$$

$$\geq \frac{3}{4}.2^2.(a^2 + b^2 + c^2)^2 \overset{Ionescu\ -\ Weltzenbock}{\geq} 3\left(4\sqrt{3}F\right)^2 = 144F^2$$

Equality holds for: $a = b = c = 1,\ u(T) = 2$.