ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\left(\frac{a}{h_b}+\frac{b}{h_c}+\frac{c}{h_a}\right)\left(\frac{1}{(a+b)^2}+\frac{1}{(b+c)^2}+\frac{1}{(c+a)^2}\right)\geq\frac{9}{8F}$$

Proposed by D.M.Bătinețu-Giurgiu, Mihaly Bencze-Romania

Solution by Tapas Das-India

$$\left(\frac{a}{h_b}+\frac{b}{h_c}+\frac{c}{h_a}\right)\left(\frac{1}{(a+b)^2}+\frac{1}{(b+c)^2}+\frac{1}{(c+a)^2}\right)=$$

$$=\left(\frac{a}{\frac{2F}{b}}+\frac{b}{\frac{2F}{c}}+\frac{c}{\frac{2F}{a}}\right)\left(\frac{1}{(a+b)^2}+\frac{1}{(b+c)^2}+\frac{1}{(c+a)^2}\right)=$$

$$=\frac{1}{2F}(ab+bc+ca)\left(\frac{1}{(a+b)^2}+\frac{1}{(b+c)^2}+\frac{1}{(c+a)^2}\right)\overset{Iran\ 1996}{\geq}\frac{1}{2F}\cdot\frac{9}{4}=\frac{9}{8F}$$

Equality holds for an equilateral triangle.