ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$a^4+b^4+c^4+abc(a+b+c)\geq 32F^2$$

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$$a^4+b^4+c^4= a^3.a+b^3.b+c^3.c\overset{Chebyshev}{\geq}\frac{1}{3}(a^3+b^3+c^3)(a+b+c)\overset{AM-GM}{\geq}$$

$$\geq\frac{1}{3}(3abc)(a+b+c)=abc(a+b+c)\ (1)$$

$$a^4+b^4+c^4+abc(a+b+c)\overset{(1)}{\geq}2abc(a+b+c)=2.4RF.2s=$$

$$=16sFR\overset{EULER}{\geq}16Fs(2r)=32F(rs)=32F^2$$

Equality holds for an equilateral triangle.