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If $m \geq 0$ then in any triangle ABC with the area F the following inequality holds:

$$\frac{a}{h_a^{2m+1}}\cdot cot^{m+1}(A)+\frac{b}{h_b^{2m+1}}\cdot cot^{m+1}(B)+\frac{c}{h_c^{2m+1}}\cdot cot^{m+1}(C)\geq\frac{2}{3^m.F^m}$$

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$$\frac{a}{h_a^{2m+1}}\cdot cot^{m+1}(A)=\frac{a}{h_a^m}\cdot\frac{cot^{m+1}(A)}{h_a^{m+1}}=\frac{a}{\left(\frac{2F}{a}\right)^m}\cdot\left(\frac{\cot(A)}{h_a}\right)^{m+1}=\frac{1}{(2F)^m}\cdot\left(\frac{a\cot(A)}{h_a}\right)^{m+1}=$$

$$=\frac{1}{(2F)^m}\cdot\left(\frac{a\cos(A)}{\frac{bc}{2R}\cdot\sin(A)}\right)^{m+1}=\frac{1}{(2F)^m}\cdot\left(\frac{Ra\cos(A)}{F}\right)^{m+1}=\frac{R^{m+1}}{2^m.F^{m+1}}\cdot\left(\frac{(a.\cos(A))^{m+1}}{F^m}\right)$$

Similarly:

$$\boxed{\begin{array}{l}\frac{b}{h_b^{2m+1}}\cdot cot^{m+1}(B)=\frac{R^{m+1}}{2^m.F^{m+1}}\cdot\left(\frac{(b.\cos(B))^{m+1}}{F^m}\right)\\ \frac{c}{h_c^{2m+1}}\cdot cot^{m+1}(C)=\frac{R^{m+1}}{2^m.F^{m+1}}\cdot\left(\frac{(c.\cos(C))^{m+1}}{F^m}\right)\end{array}}$$

Then:

$$\sum_{cyc}\frac{a}{h_a^{2m+1}}\cdot cot^{m+1}(A)=\frac{R^{m+1}}{2^m.F^{m+1}}\sum_{cyc}\frac{(a.\cos(A))^{m+1}}{F^m}\overset{Radon}{\geq}$$

$$\geq\frac{R^{m+1}}{2^m.F^{m+1}}\cdot\frac{(a.\cos(A)+b.\cos(B)+c.\cos(C))^{m+1}}{(F+F+F)^m}=\frac{R^{m+1}}{2^m.F^{m+1}}\cdot\frac{\left(\frac{2F}{R}\right)^{m+1}}{(3F)^m}=\frac{2}{3^m.F^m}$$

Equality holds for $m=0,\ a=b=c$.