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If $x,y>0$ and M an interior point in triangle ABC with the area F and

$F_a = \text{area } MBC, F_b = \text{area } MCA, F_c = \text{area } MAB$, *then* :

$$\frac{a^3b^3}{(xF_a+yF_b)^2}+\frac{b^3c^3}{(xF_b+yF_c)^2}+\frac{c^3a^3}{(xF_c+yF_a)^2}\ge\frac{192\sqrt{3}}{(x+y)^2}F$$

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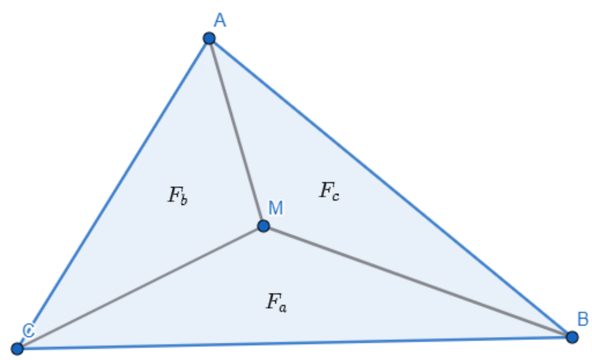
Solution by Mirsadix Muzefferov-Azerbaijan



$$F_a+F_b+F_c=F\ (1)$$

$$\frac{a^3b^3}{(xF_a+yF_b)^2}+\frac{b^3c^3}{(xF_b+yF_c)^2}+\frac{c^3a^3}{(xF_c+yF_a)^2}\ge^{Radon}$$

$$\frac{(ab+bc+ac)^3}{\big((x+y)F_a+(x+y)F_b+(x+y)F_c\big)^2}=\frac{(ab+bc+ca)^3}{(x+y)^2(F_a+F_b+F_c)^2}=^{(1)}$$

$$\frac{(ab+bc+ca)^3}{(x+y)^2F^2}\ge^{Gordon}\frac{\left(4\sqrt{3}F\right)^3}{(x+y)^2F^2}=\frac{192\sqrt{3}F}{(x+y)^2}$$

Equality holds for $a=b=c$.