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In any triangle ABC with the area F, the following inequality holds:

$$(a+2)(b+2)(c+2) \geq 18.\sqrt[4]{27}.\sqrt{F}$$

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Solution by Soumava Chakraborty-Kolkata-India

Due to Arkady Alt, if $X, Y, Z, T > 0$, *then*: $(X^2+T^2)(Y^2+T^2)(Z^2+T^2) \geq \frac{3}{4}.T^4.(X+Y+Z)^2$, with equality iff $X = Y = Z = \frac{T}{\sqrt{2}}$ *and now*,

$$(a+2)(b+2)(c+2) = (X^2+T^2)(Y^2+T^2)(Z^2+T^2)$$

$$\left(X=\sqrt{a}, Y=\sqrt{b}, Z=\sqrt{c}, T=\sqrt{2}\right) \overset{\text{Arkady Alt}}{\geq} \frac{3}{4}.\left(\sqrt{2}\right)^4.\left(\sqrt{a}+\sqrt{b}+\sqrt{c}\right)^2$$

$$\overset{\text{AM-GM}}{\geq} 27.\sqrt[3]{abc} = 27.\sqrt[3]{4\sqrt{R^2}.F} \overset{\substack{\text{Mitrinovic}\\ \text{and}\\ \text{Euler}}}{\geq} 27.\sqrt[3]{4\sqrt{\frac{2s}{3\sqrt{3}}}.2r.F}$$

$$= 54.\sqrt[3]{\frac{1}{\sqrt[4]{27}}.F\sqrt{F}} = 18.\sqrt{F}.\sqrt[3]{\frac{27}{\sqrt[4]{27}}} = 18.\sqrt{F}.\sqrt[3]{(27)^{\frac{3}{4}}} = 18.\sqrt[4]{27}.\sqrt{F}$$

$$\therefore (a+2)(b+2)(c+2) \geq 18.\sqrt[4]{27}.\sqrt{F}\ \forall\ \Delta\ ABC,$$

$$'' ='' \text{ iff } a = b = c = 1 \text{ (QED)}$$