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In ΔABC the following relationship holds:

$$\frac{a^2b}{c+a}+\frac{b^2c}{a+b}+\frac{c^2a}{b+c}\geq 2\sqrt{3}F$$

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Solution by Tapas Das-India

$$\frac{a^2b}{c+a}+\frac{b^2c}{a+b}+\frac{c^2a}{b+c}=\sum\frac{a^2b}{c+a}=\sum\frac{a^2b^2}{cb+ba}\overset{Bergstrom}{\geq}$$

$$\geq\frac{(\sum ab)^2}{2\sum ab}=\frac{1}{2}\sum ab\overset{Gordon}{\geq}\frac{1}{2}.4\sqrt{3}F=2\sqrt{3}F$$

Equalitý holds for an equilateral triangle.