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If $x, y, z > 0$ *then in* any triangle ABC with the *area* F, the following relationship holds :

$$(x^2a^4+2).(y^2b^4+2).(z^2c^4+2) \geq 48(xy+yz+zx)F^2$$

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Due to Arkady Alt, if $X, Y, Z, T > 0$, *then* : $(X^2+T^2)(Y^2+T^2)(Z^2+T^2) \geq \frac{3}{4}.T^4.(XY+YZ+ZX)^2$, with equality iff $X = Y = Z = \frac{1}{\sqrt{2}}$ *and now*,

$$(x^2a^4+2).(y^2b^4+2).(z^2c^4+2) = (X^2+T^2)(Y^2+T^2)(Z^2+T^2)$$

$$\left(X = xa^2, Y = yb^2, Z = zc^2, T = \sqrt{2}\right) \overset{\text{Arkady Alt}}{\geq} \frac{3}{4}.\left(\sqrt{2}\right)^4.(xa^2+yb^2+zc^2)^2$$

$$\overset{\text{Oppenheim}}{\geq} 3\left(4F.\sqrt{xy+yz+zx}\right)^2 \therefore (x^2a^4+2).(y^2b^4+2).(z^2c^4+2) \geq$$

$48.(xy+yz+zx)F^2\ \forall\, x, y, z > 0$ *and* $\forall\, \Delta\, ABC$, $'' = ''$ iff $xa^2 = yb^2 = zc^2 = \frac{1}{\sqrt{2}}$ *and* $a = b = c$ *and* so, $'' = ''$ iff $x = y = z = \frac{1}{\sqrt{2}.k^2}$ $(k = a = b = c)$

$\wedge\ \Delta\, ABC$ is equilateral (QED)