ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c,x,y>0$ then:

$$a^3+b^3+c^3+\frac{a}{(bx+cy)^2}+\frac{b}{(cx+ay)^2}+\frac{c}{(ax+by)^2}\ge\frac{2}{x+y}(a+b+c)$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$a^3+b^3+c^3+\frac{a}{(bx+cy)^2}+\frac{b}{(cx+ay)^2}+\frac{c}{(ax+by)^2}=$$

$$=\left(a^3+\frac{a}{(bx+cy)^2}\right)+\left(b^3+\frac{b}{(cx+ay)^2}\right)+\left(c^3+\frac{c}{(ax+by)^2}\right)\overset{A-G}{\geq}$$

$$\ge 2.\frac{a^2}{bx+cy}+2.\frac{b^2}{cx+ay}+2.\frac{c^2}{ax+by}=$$

$$=2\left(\frac{a^2}{bx+cy}+\frac{b^2}{cx+ay}+\frac{c^2}{ax+by}\right)\overset{Bergstrom}{\geq}2\frac{(a+b+c)^2}{(x+y)(a+b+c)}=\frac{2}{x+y}(a+b+c)$$