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If $a,b,c>0$ then:

$$a^3+b^3+c^3+\frac{a}{(b+c)^2}+\frac{b}{(a+c)^2}+\frac{c}{(b+a)^2}\geq(a+b+c)$$

Proposed by D.M.Bătinețu-Giurgiu, Neculai Stanciu-Romania

Solution by Mirsadix Muzefferov-Azerbaijan

$$a^3+b^3+c^3+\frac{a}{(b+c)^2}+\frac{b}{(a+c)^2}+\frac{c}{(b+a)^2}=$$
$$=a^3+b^3+c^3+\frac{a^3}{(ab+ac)^2}+\frac{b^3}{(bc+ab)^2}+\frac{c^3}{(ac+bc)^2}=$$
$$=a^3\left(1+\frac{1}{(ab+ac)^2}\right)+b^3\left(1+\frac{1}{(bc+ab)^2}\right)+c^3\left(1+\frac{1}{(ac+bc)^2}\right)\overset{A-G}{\geq}$$
$$\geq a^3.\frac{2}{(ab+ac)}+b^3.\frac{2}{(bc+ab)}+c^3.\frac{2}{(ac+bc)}=$$
$$=2\left(\frac{a^2}{b+c}+\frac{b^2}{a+c}+\frac{c^2}{a+b}\right)\overset{Bergstrom}{\geq}2.\frac{(a+b+c)^2}{2(a+b+c)}=a+b+c$$

$$Equality\ \ holds\ for\ a=b=c=\frac{1}{\sqrt{2}}$$