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If $x, y, z, r, s, t, u, v, w > 0$ then:

$$(x^2 + ru^2)(y^2 + sv^2)(z^2 + tw^2) \geq \frac{3}{4}(x\sqrt{s} \cdot \sqrt{t} \cdot vw + y\sqrt{r} \cdot \sqrt{t} \cdot uw + z\sqrt{r} \cdot \sqrt{s} \cdot uv)^2$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} & (x^2 + ru^2)(y^2 + sv^2)(z^2 + tw^2) = \\ & = (rst)(uvw)^2 \left(\frac{x^2}{ru^2} + 1 \right) \left(\frac{y^2}{sv^2} + 1 \right) \left(\frac{z^2}{tw^2} + 1 \right) \stackrel{\text{Arkady Alt's}}{\geq} \\ & \geq \frac{3}{4} \cdot 1^4 \cdot \left(\frac{x}{\sqrt{ru}} + \frac{y}{\sqrt{sv}} + \frac{z}{\sqrt{tw}} \right)^2 \cdot (rst)(uvw)^2 = \\ & = \frac{3}{4} (x\sqrt{s} \cdot \sqrt{t} \cdot vw + y\sqrt{r} \cdot \sqrt{t} \cdot uw + z\sqrt{r} \cdot \sqrt{s} \cdot uv)^2 \end{aligned}$$

$$\text{Equality holds for : } \frac{x}{\sqrt{ru}} = \frac{y}{\sqrt{sv}} = \frac{z}{\sqrt{tw}} = \frac{1}{\sqrt{2}}$$