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If $a, b, c > 0$, *then prove that* :

$$\sum_{cyc} \frac{a}{\sqrt{4b^2 + bc + 4c^2}} \ge 1$$

Proposed by D.M. Bătinețu-Giurgiu, Neculai Stanciu-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc} \frac{a}{\sqrt{4b^2 + bc + 4c^2}} = \sum_{cyc} \frac{a^{\frac{3}{2}}}{\sqrt{4ab^2 + abc + 4c^2a}} \overset{\text{Radon}}{\ge}$$

$$\frac{\left(\sum_{cyc} a\right)^{\frac{3}{2}}}{\sqrt{3abc + 4\sum_{cyc} a^2b + 4\sum_{cyc} ab^2}} \overset{?}{\ge} 1 \Leftrightarrow \left(\sum_{cyc} a\right)^3 \overset{?}{\ge} 3abc + 4\sum_{cyc} a^2b + 4\sum_{cyc} ab^2$$

$$\Leftrightarrow \sum_{cyc} a^3 + 3\left(2abc + \sum_{cyc} a^2b + \sum_{cyc} ab^2\right) \overset{?}{\ge} 3abc + 4\sum_{cyc} a^2b + 4\sum_{cyc} ab^2$$

$$\Leftrightarrow \sum_{cyc} a^3 + 3abc \overset{?}{\ge} \sum_{cyc} a^2b + \sum_{cyc} ab^2 \to \text{true via Schur}$$

$$\therefore \sum_{cyc} \frac{a}{\sqrt{4b^2 + bc + 4c^2}} \ge 1 \ \forall\, a, b, c > 0, '' ='' \text{ iff } a = b = c \ (\text{QED})$$