ROMANIAN MATHEMATICAL MAGAZINE

$$Let\ \ be\ \ f:\ R_+^* \times R_+^* \to\ R_+^*,\qquad then:$$

$$\left(\frac{x^2}{\left(y+f(x,y)\right)^2}+2\right)\cdot\left(\frac{y^2}{\left(y+f(x,y)\right)^2}+2\right)\cdot\left(\frac{\left(f(x,y)\right)^2}{(y+x)^2}+2\right)\geq\frac{27}{4}\ \ \forall x,y\in R_+^*$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$According\ \ to:$$

$$\boxed{\forall x,y,z,t>0\ \ \left(x^2+t^2\right)\left(y^2+t^2\right)\left(z^2+t^2\right)\geq\frac{3}{4}t^4(x+y+z)^2}$$

$$formula\qquad\qquad\qquad\qquad(Arkady\ \ Alt)$$

$$\frac{x}{y+f(x,y)}=a,\qquad \frac{y}{x+f(x,y)}=b,\qquad \frac{f(x,y)}{x+y}=c,\qquad t^2=2$$

$$Then:\ \ \left(a^2+2\right)\left(b^2+2\right)\left(c^2+2\right)\geq\frac{3}{4}.4(a+b+c)^2=$$

$$=3\left(\frac{x}{y+f(x,y)}+\frac{y}{x+f(x,y)}+\frac{f(x,y)}{x+y}\right)^2=$$

$$=3\left(\frac{x^2}{xy+xf(x,y)}+\frac{y^2}{xy+yf(x,y)}+\frac{f^2(x,y)}{f(x,y)(x+y)}\right)^2\overset{Bergstrom}{\geq}$$

$$\geq 3\left(\frac{\left(x+y+f(x,y)\right)^2}{2(xy+xf+yf)}\right)^2=\frac{3}{4}\left(\frac{(x+y+f)^2}{(xy+xf+yf)}\right)^2\geq\frac{3}{4}.3^2=\frac{27}{4}$$