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If $a, b, c, s, t, u > 0$, *then* :

$$(a^2+s^2)(b^2+t^2)(c^2+u^2) \geq \frac{3}{4}(atu+bsu+cst)^2$$

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Due to Arkady Alt, if $x, y, z, T > 0$, *then* :

$$(x^2+T^2)(y^2+T^2)(z^2+T^2) \geq \frac{3}{4}.T^4.(xy+yz+zx)^2,$$

with equality iff $x = y = z = \frac{1}{\sqrt{2}} \to (1)$

$$(a^2+s^2)(b^2+t^2)(c^2+u^2) =$$

$$= \frac{(a^2t^2u^2+s^2t^2u^2)(b^2s^2u^2+s^2t^2u^2)(c^2s^2t^2+s^2t^2u^2)}{(stu)^4} =$$

$$= \frac{(x^2+T^2)(y^2+T^2)(z^2+T^2)}{(stu)^4}\ (x = atu, y = bsu, z = cst, T = stu) \geq$$

$$\overset{\text{Arkady Alt}}{\geq} \frac{\frac{3}{4}.T^4.(xy+yz+zx)^2}{(stu)^4} = \frac{\frac{3}{4}.(stu)^4.(atu+bsu+cst)^2}{(stu)^4}$$

$$\therefore (a^2+s^2)(b^2+t^2)(c^2+u^2) \geq \frac{3}{4}(atu+bsu+cst)^2\ \forall\ a, b, c, s, t, u > 0,$$

$$'' ='' \text{ iff } atu = bsu = cst = \frac{1}{\sqrt{2}}\ \text{(QED)}$$