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If $m \geq 0$ *and* $a, b, c, x, y > 0$ *and* $abc = 1$ *then*:

$$\frac{1}{a^{2m+1}(bx+cy)^m}+\frac{1}{b^{2m+1}(cx+ay)^m}+\frac{1}{c^{2m+1}(ax+by)^m}\geq\frac{3}{(x+y)^m}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{1}{a^{2m+1}(bx+cy)^m}+\frac{1}{b^{2m+1}(cx+ay)^m}+\frac{1}{c^{2m+1}(ax+by)^m}=$$

$$=\frac{\frac{1}{a^{m+1}}}{a^m(bx+cy)^m}+\frac{\frac{1}{b^{m+1}}}{b^m(cx+ay)^m}+\frac{\frac{1}{c^{m+1}}}{c^m(ax+by)^m}=$$

$$=\frac{\left(\frac{1}{a}\right)^{m+1}}{(abx+acy)^m}+\frac{\left(\frac{1}{b}\right)^{m+1}}{(bcx+aby)^m}+\frac{\left(\frac{1}{c}\right)^{m+1}}{(acx+bcy)^m}\overset{Radon}{\geq}\frac{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^{m+1}}{\left((ab+bc+ac)(x+y)\right)^m}=$$

$$=\frac{(ab+bc+ac)^{m+1}}{(ab+bc+ac)^m(abc)^{m+1}(x+y)^m}=\frac{ab+bc+ac}{(x+y)^m}\overset{AM-GM}{\geq}\frac{3((abc)^2)^{\frac{1}{3}}}{(x+y)^m}=\frac{3}{(x+y)^m}$$

$$Equality\ holds\ for:\ a=b=c=1.$$