ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ and $n\in\mathbb{N}$ then prove that :

$$\frac{a+b}{(b+c)^n}+\frac{b+c}{(c+a)^n}+\frac{c+a}{(a+b)^n}\geq\frac{(a+b+c)^{n+1}}{2^{n-1}(a^2+b^2+c^2)^n}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Soumava Chakraborty-Kolkata-India

$$LHS\overset{AM-GM}{\geq}\frac{3}{\left(\sqrt[3]{\prod_{cyc}(b+c)}\right)^{n-1}}\overset{AM-GM}{\geq}\frac{3}{\left(\frac{2\sum_{cyc}a}{3}\right)^{n-1}}=\frac{3^n\left(\sum_{cyc}a\right)^{n+1}}{2^{n-1}\left(\left(\sum_{cyc}a\right)^2\right)^n}=$$

$$\overset{CBS}{\geq}\frac{3^n\left(\sum_{cyc}a\right)^{n+1}}{2^{n-1}\left(3\sum_{cyc}a^2\right)^n}=\frac{(a+b+c)^{n+1}}{2^{n-1}(a^2+b^2+c^2)^n},''='' \text{ iff } a=b=c \text{ (QED)}$$