ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$, $n\geq 2$ then:

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\geq\frac{(a+b+c)^{n+1}}{\left(a\sqrt[n]{b+c}+b\sqrt[n]{c+a}+c\sqrt[n]{a+b}\right)^{n}}$$

Proposed by Zazazhavanadze-Georgia

Solution by Tapas Das-India

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}=\sum\frac{a}{b+c}=\sum\frac{a^{n+1}}{a^{n}(b+c)}$$

$$\left(\sum\frac{a^{n+1}}{a^{n}(b+c)}\right)\left(\sum a\sqrt[n]{b+c}\right)^{n}\overset{Holder}{\geq}\left(\sum\sqrt[n+1]{\frac{a^{n+1}}{a^{n}(b+c)}\cdot a^{n}(b+c)}\right)^{n+1}=(a+b+c)^{n+1}$$

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}=\sum\frac{a}{b+c}=\sum\frac{a^{n+1}}{a^{n}(b+c)}\geq$$

$$\geq\frac{(a+b+c)^{n+1}}{\left(\sum a\sqrt[n]{b+c}\right)^{n}}=\frac{(a+b+c)^{n+1}}{\left(a\sqrt[n]{b+c}+b\sqrt[n]{c+a}+c\sqrt[n]{a+b}\right)^{n}}$$

Equality holds for a=b=c.