ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\frac{1}{(a+b)^3}+\frac{1}{(b+c)^3}+\frac{1}{(c+a)^3}\geq\frac{9(24abc)^4}{[(a+b)^3+(b+c)^3+(c+a)^3]^5}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$\frac{1}{(a+b)^3}+\frac{1}{(b+c)^3}+\frac{1}{(c+a)^3}=\sum\frac{1}{(a+b)^3}=$$

$$=\sum\frac{(a+b)^{12}}{(a+b)^{15}}=\sum\frac{((a+b)^2)^6}{((a+b)^3)^5}\overset{Radon}{\geq}\frac{(\sum(a+b)^2)^6}{(\sum(a+b)^3)^5}\geq$$

$$\overset{AM-GM}{\geq}\frac{(\sum 4ab)^6}{(\sum(a+b)^3)^5}\overset{AM-GM}{\geq}\frac{\left(12\,(a^2b^2c^2)^{\frac{1}{3}}\right)^6}{(\sum(a+b)^3)^5}=\frac{12^6(abc)^4}{(\sum(a+b)^3)^5}=$$

$$=9.\frac{(3.8.abc)^4}{(\sum(a+b)^3)^5}=\frac{9(24abc)^4}{[(a+b)^3+(b+c)^3+(c+a)^3]^5}$$

Equality holds for a=b=c.