ROMANIAN MATHEMATICAL MAGAZINE

Let a, b, c be real numbers. Prove that :

$$\frac{1}{a^2-ab+b^2}+\frac{1}{b^2-bc+c^2}+\frac{1}{c^2-ca+a^2}\geq\frac{4(a^2+b^2+c^2+ab+bc+ca)}{(a^2+b^2+c^2)^2}$$

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Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

Let $x:=a^2+b^2+c^2, y:=ab+bc+ca$.

By CBS inequality, we have

$$\frac{1}{a^2-ab+b^2}+\frac{1}{b^2-bc+c^2}+\frac{1}{c^2-ca+a^2}\geq\frac{9}{2(a^2+b^2+c^2)-(ab+bc+ca)}=$$

$$=\frac{9}{2x-y}\overset{?}{\geq}\frac{4(x+y)}{x^2}\Leftrightarrow(x-2y)^2\geq 0,$$

which is true. Equality holds iff $a=b, c=0$ and its permutation.