ROMANIAN MATHEMATICAL MAGAZINE

Let a, b, c be real numbers with $a+b+c=3$. Prove that :

$$\frac{a-1}{b^2+bc+c^2}+\frac{b-1}{c^2+ca+a^2}+\frac{c-1}{a^2+ab+b^2}\geq 0$$

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Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

$$3\sum_{cyc}\frac{a-1}{b^2+bc+c^2}=\sum_{cyc}\frac{2a-b-c}{b^2+bc+c^2}=$$

$$=\sum_{cyc}\left(\frac{a-b}{b^2+bc+c^2}-\frac{c-a}{b^2+bc+c^2}\right)=$$

$$=\sum_{cyc}\left(\frac{b-c}{c^2+ca+a^2}-\frac{b-c}{a^2+ab+b^2}\right)=\sum_{cyc}\frac{(a+b+c)(b-c)^2}{(a^2+ab+b^2)(c^2+ca+a^2)}\geq 0$$

Equaliy holds iff $a=b=c=1$.