ROMANIAN MATHEMATICAL MAGAZINE

Let $a,b,c\geq 0, a+b+c=1$. Prove that :

$$a\sqrt{a^2+4b}+b\sqrt{b^2+4c}+c\sqrt{c^2+4a}\geq 1$$

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Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

$$a\sqrt{a^2+4b}+b\sqrt{b^2+4c}+c\sqrt{c^2+4a}\geq$$

$$\geq a\sqrt{a^2+4b(a+b)}+b\sqrt{b^2+4c(b+c)}+c\sqrt{c^2+4a(c+a)}$$

$$=a(a+2b)+b(b+2c)+c(c+2a)=(a+b+c)^2=1.$$

Equality holds iff $abc=0$ and $a+b+c=1$.