ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, ab+bc+ca=3$ then :

$$\frac{a^2}{a+2b^2}+\frac{b^2}{b+2c^2}+\frac{c^2}{c+2a^2}\geq 2$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\frac{a^2}{a+2b^2}+\frac{b^2}{b+2c^2}+\frac{c^2}{c+2a^2}=\sum\frac{a^2}{a+2b^2}=$$

$$=\sum\left(a-\frac{2ab^2}{a+b^2+b^2}\right)\overset{AM-GM}{\geq}\sum a-\sum\frac{2ab^2}{3\sqrt[3]{ab^4}}=$$

$$=\sqrt{\left(\sum a\right)^2}-\frac{2}{3}\sum(ab)^{\frac{2}{3}}\overset{CBS}{\geq}\sqrt{3(ab+bc+ca)}-\frac{2}{3}.3\left(\frac{ab+bc+ca}{3}\right)^{\frac{2}{3}}=$$

$$\overset{ab+bc+ca=3}{=}3-1=2$$

Equality holds for $a=b=c=1$.