ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z>0,\ x+y+xy=x^2+y^2$ then:

$$\frac{x}{y+2xy}+\frac{y}{x+2xy}\geq\frac{2}{5}$$

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$$x+y+xy=x^2+y^2$$

$$(x+y)+xy=(x+y)^2-2xy$$

$$(x+y)+3xy=(x+y)^2$$

$$(x+y)+3\frac{(x+y)^2}{4}\overset{AM-GM}{\geq}(x+y)^2$$

$$(x+y)\geq\frac{1}{4}(x+y)^2$$

$$4\geq(x+y)$$

$$(x+y)\leq 4\ (1)$$

$$\frac{x}{y+2xy}+\frac{y}{x+2xy}=\frac{x^2}{xy+2x^2y}+\frac{y^2}{xy+2xy^2}\overset{Bergstrom}{\geq}$$

$$\geq\frac{(x+y)^2}{2xy+2xy(x+y)}\overset{AM-GM}{\geq}\frac{(x+y)^2}{2\frac{(x+y)^2}{4}+2\frac{(x+y)^2}{4}(x+y)}=\frac{1}{\frac{1}{2}+\frac{x+y}{2}}\overset{(1)}{\geq}\frac{1}{\frac{1}{2}+\frac{1}{2}.4}=\frac{2}{5}$$

Equality holds for $x=1,y=2$ or $x=2,y=1$.