ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\geq 2-\frac{ab+bc+ca}{2(a^2+b^2+c^2)}$$

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Solution by Tapas Das-India

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}=\sum\frac{a}{b+c}=\sum\frac{a^2}{ab+ac}\overset{CBS}{\geq}\frac{(a+b+c)^2}{2(ab+bc+ca)}=$$

$$=\frac{a^2+b^2+c^2+2(ab+bc+ca)}{2(ab+bc+ca)}\overset{\frac{a^2+b^2+c^2}{ab+bc+ca}=x\geq 1}{=}\frac{x}{2}+1$$

$$We\ need\ to\ \ show:$$

$$\frac{x}{2}+1\geq 2-\frac{ab+bc+ca}{2(a^2+b^2+c^2)}\overset{\frac{a^2+b^2+c^2}{ab+bc+ca}=x\geq 1}{=}2-\frac{1}{2x}$$

$$\frac{x+2}{2}\geq\frac{4x-1}{2x}$$

$$x^2-2x+1\geq 0$$

$$(x-1)^2\geq 0\ true$$

Equality holds for $a=b=c$.