ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\ a^2+b^2+c^2=1$ then:

$$\frac{\sqrt{3a^2+4ab+3b^2}}{ab}+\frac{\sqrt{3b^2+4bc+3c^2}}{bc}+\frac{\sqrt{3c^2+4ca+3a^2}}{ca}\geq 3\sqrt{30}$$

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Solution by Tapas Das-India

$$3a^2+4ab+3b^2=3(a^2+b^2)+4ab\overset{AM-GM}{\geq}6ab+4ab=10ab\ (1)$$

$$\frac{\sqrt{3a^2+4ab+3b^2}}{ab}+\frac{\sqrt{3b^2+4bc+3c^2}}{bc}+\frac{\sqrt{3c^2+4ca+3a^2}}{ca}=$$

$$=\sum\frac{\sqrt{3a^2+4ab+3b^2}}{ab}\overset{(1)}{\geq}\sum\frac{\sqrt{10ab}}{ab}=$$

$$=\sqrt{10}\sum\frac{1}{\sqrt{ab}}\overset{CBS}{\geq}\frac{\sqrt{10}\,(1+1+1)^2}{\sqrt{ab}+\sqrt{bc}+\sqrt{ca}}\overset{CBS}{\geq}\frac{9\sqrt{10}}{\sqrt{3(ab+bc+ca)}}\geq$$

$$\geq\frac{9\sqrt{10}}{\sqrt{3(a^2+b^2+c^2)}}\overset{a^2+b^2+c^2=1}{=}\frac{9\sqrt{10}}{\sqrt{3}}=3\sqrt{30}$$

Equality holds for $a=b=c=1$.