ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, a+b+c=\frac{3}{2}$ then:

$$\frac{1+b}{1+4a^2}+\frac{1+c}{1+4b^2}+\frac{1+a}{1+4c^2}\geq\frac{9}{4}$$

*Proposed by Nguyen Hung Cuong-Vietnam*

*Solution by Tapas Das-India*

$$\frac{1+b}{1+4a^2}=(1+b)-\frac{4a^2(1+b)}{1+4a^2}\overset{AM-GM}{\geq}(1+b)-\frac{4a^2(1+b)}{4a}=$$

$$=(1+b)-a(1+b)=1+b-a-ab\ (1)$$

$$\frac{1+b}{1+4a^2}+\frac{1+c}{1+4b^2}+\frac{1+a}{1+4c^2}=\sum\frac{1+b}{1+4a^2}\overset{(1)}{\geq}\sum(1+b-a-ab)$$

$$=3+\sum b-\sum a-\sum ab\geq 3-\frac{(\sum a)^2}{3}=3-\frac{\left(\frac{3}{2}\right)^2}{3}=3-\frac{3}{4}=\frac{9}{4}$$

$$Equality\ holds\ for\ a=b=c=\frac{1}{2}.$$