ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$, $a^2b^2+b^2c^2+c^2a^2=12$ then:

$$c\sqrt[3]{ab^2}+a\sqrt[3]{bc^2}+b\sqrt[3]{ca^2}\leq 6$$

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Solution by Tapas Das-India

$$\sum ab=\sqrt{\left(\sum ab\right)^2}\overset{CBS}{\leq}\sqrt{3\sum a^2b^2}\overset{\sum a^2b^2=12}{=}\sqrt{3\times 12}=6(1)$$

$$c\sqrt[3]{ab^2}+a\sqrt[3]{bc^2}+b\sqrt[3]{ca^2}=\sum c\sqrt[3]{ab^2}=\sum\sqrt[3]{ac^3b^2}=$$

$$=\sum\sqrt[3]{ac.bc.bc}\overset{AM-GM}{\leq}\sum\frac{ac+bc+bc}{3}=$$

$$=\frac{2}{3}\sum bc+\frac{1}{3}\sum ac\overset{(1)}{\leq}\frac{2}{3}\times 6+\frac{1}{3}\times 6=6$$

$$Equality\ holds\ for\ a=b=c=\sqrt{2}.$$