ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z \in [0, 1]$ then:

$$\frac{1}{2-x}+\frac{1}{2-y}+\frac{1}{2-z}-3xyz \geq 0$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

Let be:

$$f:[0,1]X[0,1]X[0,1] \to \mathbb{R}, f(x,y,z)=\frac{1}{2-x}+\frac{1}{2-y}+\frac{1}{2-z}-3xyz$$

$$f'_x=\frac{1}{(2-x)^2}-3yz, \qquad f''(x)=\frac{2}{(2-x)^3}>0$$

Analogous:

$$f''(y)>0, f''(z)>0.$$

f −convex in each variable on the cube $[0,1]X[0,1]X[0,1]$.

$$minf(x,y,z)=min\begin{Bmatrix} f(0,0,0), f(1,0,0), f(0,1,0), f(0,0,1), f(0,1,0), \\ f(0,0,1), f(1,1,0), f(1,0,1), f(0,1,1), f(1,1,1) \end{Bmatrix}=$$

$$=min\left\{\frac{3}{2}, 2, \frac{5}{2}, 0\right\}=0 \Rightarrow minf(x,y,z)=0 \Rightarrow$$

$$\frac{1}{2-x}+\frac{1}{2-y}+\frac{1}{2-z}-3xyz \geq 0$$