ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}\geq\frac{(a+b+c)^2}{2\sqrt{3(ab+bc+ca)}}$$

*Proposed by Nguyen Hung Cuong-Vietnam*

*Solution by Tapas Das-India*

$$(ab+bc+ca)=\sqrt{(ab+bc+ca)^2}=\sqrt{(ab+bc+ca)(ab+bc+ca)}\leq$$

$$\leq\sqrt{\frac{(ab+bc+ca)(a+b+c)^2}{3}}=\frac{a+b+c}{\sqrt{3}}\sqrt{ab+bc+ca}\,(1)$$

$$\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}=\sum\frac{a^2}{b+c}=\sum\frac{a^3}{ab+ac}\overset{Holder}{\geq}\frac{(a+b+c)^3}{3(2ab+2bc+2ca)}\overset{(1)}{\geq}$$

$$\geq\frac{(a+b+c)^3}{6.\frac{a+b+c}{\sqrt{3}}\sqrt{ab+bc+ca}}\geq\frac{(a+b+c)^2}{2\sqrt{3(ab+bc+ca)}}$$

Equality holds for $a=b=c=1$.