ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\frac{a^3+2a^2b}{a^2+3ab+b^2}+\frac{b^3+2b^2c}{b^2+3bc+c^2}+\frac{c^3+2c^2a}{c^2+3ca+a^2}\geq\frac{3(a+b+c)}{5}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\frac{a^3+2a^2b}{a^2+3ab+b^2}+\frac{b^3+2b^2c}{b^2+3bc+c^2}+\frac{c^3+2c^2a}{c^2+3ca+a^2}=$$

$$=\sum\frac{a^3+2a^2b}{a^2+3ab+b^2}=\sum\frac{a^2(a+2b)}{a(a+2b)+b(a+b)}=$$

$$=\sum\left(a-\frac{ab(a+b)}{a(a+2b)+b(a+b)}\right)=$$

$$=\sum a-\sum\frac{ab(a+b)}{a^2+3ab+b^2}\overset{AM-GM}{\geq}\sum a-\sum\frac{ab(a+b)}{2ab+3ab}=$$

$$=\sum a-\frac{\sum(a+b)}{5}=\frac{3(a+b+c)}{5}$$

Equality holds for $a=b=c$.