ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$, $x + y + z = 3$ then:

$$\sum \frac{x^3}{x^2 + y^2} \geq \frac{3}{2}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\sum \frac{x^3}{x^2 + y^2} = \sum \left(x - \frac{xy^2}{x^2 + y^2} \right) \overset{AM-GM}{\geq} \sum \left(x - \frac{xy^2}{2xy} \right) =$$

$$= \sum \left(x - \frac{y}{2} \right) = \frac{\sum x}{2} = \frac{3}{2}$$

$Equality\ holds\ for\ x = y = z = 1.$